

# Universality of the saturation scale and the initial eccentricity in heavy ion collisions

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Recent estimates that Color Glass Condensate initial conditions may generate a larger initial eccentricity for noncentral relativistic heavy ion collisions (relative to the initial eccentricity assumed in earlier hydrodynamic calculations) have raised the possibility of a higher bound on the viscosity of the Quark Gluon Plasma. We show that this large initial eccentricity results in part from a definition of the saturation scale as proportional to the number of nucleons participating in the collision. A saturation scale proportional to the nuclear thickness function (and therefore independent of the probe) leads to a smaller eccentricity, albeit still larger than the value used in hydrodynamic models. Our results suggest that the early elliptic flow in heavy ion collisions (unlike multiplicity distributions) is sensitive to the universality of the saturation scale in high energy QCD.

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## I. INTRODUCTION

Data from RHIC on bulk properties [1, 2, 3, 4] of the matter produced in heavy ion collisions, is qualitatively described by hydrodynamic calculations [5]. Increasingly precise experimental data now enables us to probe details of initial conditions for the hydrodynamical calculations. These initial conditions may be obtained from the Color Glass Condensate (CGC) framework [6] describing the phenomenon of parton saturation [7, 8] in the high energy nuclear wavefunction.

One particular application of the CGC framework to the early stages of heavy ion collisions has been the formulation of the problem as a solution of classical field equations [9, 10, 11, 12, 13] and the numerical solution of these equations [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. By solving classical equations, one efficiently sums all leading order Feynman graphs contributing to the inclusive multiplicity. We shall henceforth refer to this approach as the Classical Yang-Mills (CYM) approach. An alternative approach is to assume the framework of  $k_\perp$  factorization, where the inclusive multiplicity of produced gluons is expressed as a convolution of unintegrated gluon distributions from the projectile and the target. While exact to leading order for proton-nucleus collisions [26, 27, 28], this approach is only approximate in heavy ion collisions at central rapidities. A popular version of the  $k_\perp$  factorization approach applied to heavy ion collisions, with a particular ansatz for the unintegrated gluon distributions, is the KLN approach [29, 30, 31, 32, 33]. The solution of the CYM equations incorporates, to lowest order in the coupling, all terms that violate  $k_\perp$  factorization; these solutions however are numerically intensive. On the other hand, the KLN ansatz, while approximate, provides an analytic expression that captures some key features of saturation.

A striking signal of collective hydrodynamical behavior of the matter produced at RHIC is elliptic flow [34, 35, 36, 37]. The large elliptic flow observed at RHIC requires strong interactions during the first fermis of the collision. The elliptic flow in this strongly interacting

system is particularly sensitive to the initial eccentricity (the anisotropy of the energy density in the transverse plane),

$$\epsilon = \frac{\int d^2\mathbf{x}_T \varepsilon(\mathbf{x}_T) (y^2 - x^2)}{\int d^2\mathbf{x}_T \varepsilon(\mathbf{x}_T) (y^2 + x^2)}, \quad (1)$$

where  $\varepsilon$  is the local energy density<sup>1</sup>. In a hydrodynamical description of heavy ion collisions, the eccentricity of the initial condition is the primary factor determining the elliptic flow observed in the final state [39].

In this short note, we will study how the eccentricity is computed in the CGC framework in the CYM and KLN models. We shall first discuss different ways used in the literature to understand the transverse coordinate dependence of the saturation scale, in particular based on its *universality*. We shall then explain how these lead to different results for  $\epsilon$ , in particular the recent results in Refs. [40, 41]. (See also Ref. [42] for a discussion of related issues for uranium-uranium collisions.)

## II. A UNIVERSAL SATURATION SCALE

In the CGC framework, the production of particles in the initial stage of a heavy ion collision is controlled by one parameter, the saturation scale  $Q_s$ . The initial spatial anisotropy of the energy density must thus come from the transverse coordinate dependence of the saturation scale. In the original MV model [43, 44, 45] the large  $x$  degrees of freedom are thought of as independent static (independent of light cone time) color charges with a transverse color charge density  $g^2\mu^2$ . The density of color charges is a property of the nuclear wave-

<sup>1</sup> Alternately, the eccentricity is defined as the anisotropy of the entropy density in some works [38]. As we will see, this definition is less sensitive to different CGC initial conditions. It is however not the definition used in comparisons of detailed hydrodynamic models to RHIC data.

function and independent of the probe — it is universal [46]. If the color charge density arises as a superposition of independent large  $x$  partons, it is natural that the density should be proportional to the nuclear thickness function  $T_A$ , thereby providing an average measure of the number of these fast partons. It was later shown [28, 47, 48, 49, 50, 51, 52] that the MV model does indeed exhibit saturation, with  $Q_s \sim g^2\mu$ , and thus also the saturation scale  $Q_s$  should be universal with

$$Q_s^2(\mathbf{x}_T) = Q_{s0}^2 \frac{\pi R_A^2}{A} T_A(\mathbf{x}_T). \quad (2)$$

Here  $Q_{s0}$  is the average saturation scale,  $T_A(\mathbf{x}_T) = \int_{-\infty}^{\infty} dz \rho(\mathbf{r})$  is the thickness function,  $\mathbf{x}_T$  is the coordinate relative to the center of the nucleus and  $\rho(\mathbf{r})$  is the Woods-Saxon nuclear density profile, normalized as  $\int d^3\mathbf{r} \rho(\mathbf{r}) = A$ . It has also been argued [53] that at sufficiently high energy the dependence of the saturation scale on  $T_A$  and thus  $A$  would change due to high energy evolution, but we will not consider this possibility here.

The classical field picture of the MV model, with this universally defined saturation scale, was applied in both analytical [9, 10, 11, 12, 13] and numerical classical Yang-Mills (CYM) [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25] computations of the “Glasma” [54] fields in the initial stages of the collision. Noncentral collisions with finite impact parameter were studied in Refs. [19, 21] and particularly in the detailed study of Ref. [20].

In the “KLN” approach, [29, 30, 31, 32, 33], particle production is computed using unintegrated gluon distribution functions depending on the saturation scale  $Q_s(x, \mathbf{x}_T)$ . These calculations used a saturation scale dependent on the number of participant nuclei:

$$Q_s^{A^2}(\mathbf{x}_T) \sim N_{\text{part}}^A(\mathbf{x}_T), \quad (3)$$

with

$$N_{\text{part}}^A(\mathbf{x}_T) = T_A(\mathbf{x}_T) \times \left( 1 - \left( 1 - \sigma_{NN} \frac{T_B(\mathbf{x}_T - \mathbf{b}_T)}{B} \right)^B \right). \quad (4)$$

Note that as this form involves the thickness functions of both nuclei  $A$  and  $B$ , it is not manifestly universal. While a nonuniversal saturation scale has been used in many phenomenological studies, such as the EKRT “final state saturation” model of Ref. [55], the logic for introducing the second term in the original KLN works was not the final state picture. The  $N_{\text{part}}$  definition was used instead to model the interaction probability for the scattering in a manner as close to the original Glauber framework as possible. There arises however the danger of double counting the interaction probability because this probability is already taken into account in the derivation of  $k_\perp$  factorization [26, 27, 28]. The dangers of overcounting are avoided in the universal prescription for  $Q_s$  in Eq. (2).

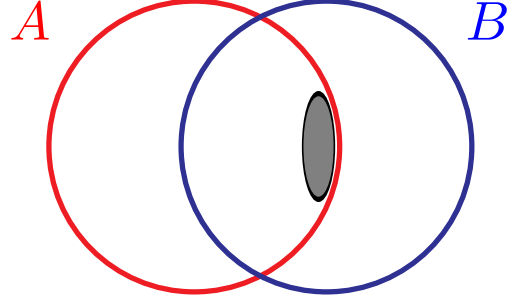


FIG. 1: The area in the transverse plane that is most important for our discussion here is marked with the gray oval, near the edge of nucleus  $A$  and the center of nucleus  $B$ . The same discussion applies for the edge of nucleus  $B$  near the center of nucleus  $A$ .

We must emphasize that the choice between using the  $k_\perp$  factorized formalism and solving the classical field equations does not dictate how the saturation scale depends on the transverse coordinate. There is no technical impediment to using  $(Q_s^A)^2 \sim T_A$  in the  $k_\perp$  factorized formulation or  $(Q_s^A)^2 \sim N_{\text{part}}^A$  in the classical field computation. In the numerical study using the classical field equations in Ref. [20] both prescriptions, Eqs. (2) and (3) were considered and were found to give similar results for the impact parameter dependence of the multiplicity. However, as we shall show in the following, they do yield different results for the eccentricity.

### III. HOW THE CHOICE OF $Q_s$ INFLUENCES THE ECCENTRICITY

For computing the eccentricity, the crucial contributions (in the geometry of the transverse plane of the scattering) come from regions where the saturation scale in one nucleus is significantly larger than in the other. Let us first review what the spectrum of gluons looks like in this case, with two saturation scales  $Q_{s1}$  and  $Q_{s2}$  satisfying  $Q_{s1} < Q_{s2}$  [13, 28]. Parametrically, the spectrum of produced gluons behaves as

$$\frac{dN}{d^2\mathbf{x}_T d^2\mathbf{p}_T} \sim \ln(p_T), \quad p_T < Q_{s1} \quad (5)$$

$$\sim \frac{Q_{s1}^2}{p_T^2}, \quad Q_{s1} < p_T < Q_{s2} \quad (6)$$

$$\sim \frac{Q_{s1}^2 Q_{s2}^2}{p_T^4}, \quad p_T > Q_{s2}. \quad (7)$$

Integrated over transverse momenta, this gives

$$\frac{dN}{d^2\mathbf{x}_T} \sim Q_{s1}^2 \quad (8)$$

$$\frac{dE_T}{d^2\mathbf{x}_T} \sim Q_{s1}^2 Q_{s2}^2, \quad (9)$$

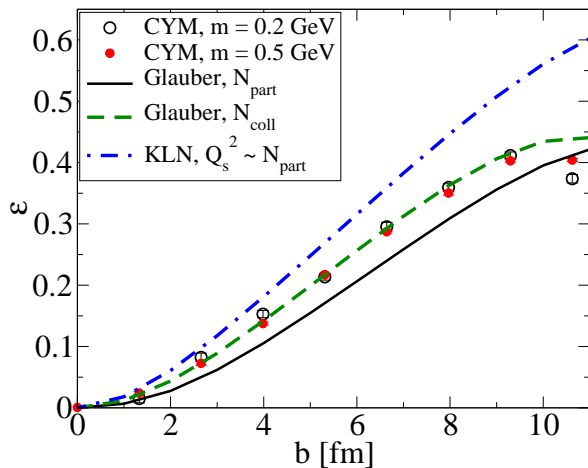


FIG. 2: The eccentricity as a function of impact parameter. The classical field CGC result with two different infrared cut-offs  $m$  is denoted by CYM. The traditional initial eccentricity used in hydrodynamics is a linear combination of mostly “Glauber  $N_{\text{part}}$ ” and a small amount of “Glauber  $N_{\text{coll}}$ ”. The “KLN” curve is the eccentricity obtained from the CGC calculation in Refs. [40, 41].

neglecting logarithmic corrections  $\sim \ln(Q_{s2}/Q_{s1})$ . The additional dependence on  $Q_{s2}$  in the transverse energy, relative to the multiplicity, holds the key to the following discussion.

The difference between the two definitions of the transverse coordinate dependence of the saturation scale, Eqs. (2) and (3) is the largest in the region near the edge of one nucleus (labeled as nucleus  $A$ ) and in the center of the other (nucleus  $B$ ), so that  $Q_s^A < Q_s^B$ ; the geometry is illustrated in Fig. 1. The smaller saturation scale approaches zero as  $(Q_s^A)^2 \sim T_A$  regardless of the definition of  $Q_s$  (Eq. (2) or Eq. (3)). But the behavior of the larger saturation scale  $Q_s^B$  is different in the two cases. Using the universal definition of  $Q_s$  in Eq. (2)  $Q_s^B$  is large,  $(Q_s^B)^2 \sim T_B$ . In contrast, the non-universal  $N_{\text{part}}$ -definition of  $Q_s$  in Eq. (3) suggests that the larger saturation scale  $Q_s^B$  also approaches zero as  $\sigma_{NN}T_A T_B$ .

Because the multiplicity (8) only depends on the smaller saturation scale  $Q_s^A$ , the difference in the gluon multiplicities between the two definitions Eqs. (2) and (3) is small. This explains the numerical observation in Ref. [20] that both the KLN prescription for  $Q_s$  and the universal CYM one give very similar results for the centrality dependence of the multiplicity. The larger saturation scale  $Q_s^B$  and therefore the energy density are, however, very different in the two cases. This difference is accentuated in the eccentricity (1). With the  $N_{\text{part}}$  definition (3), the energy density in this edge region is suppressed relative to the universal definition in (2), thereby leading to a larger eccentricity.

The eccentricities obtained using the different transverse coordinate dependences of the saturation scales are

shown in Fig. 2. The CYM eccentricity in the plot is calculated at  $\tau = 0.25$  fm, while the KLN result does not depend on time. The KLN  $N_{\text{part}}$  definition of  $Q_s$  leads to the largest eccentricity. The universal CYM definition gives smaller values of  $\epsilon$  albeit larger than the traditional parametrization (used in hydrodynamical model computations) where the energy density is taken to be proportional to the number of participating nucleons. This result is also shown to be insensitive to two different choices of the infrared scale  $m$  which regulates the spatial extent of the Coulomb tails of the gluon distribution. We observe that the values of  $\epsilon$  from the CYM computation are close to those obtained from an energy density parametrization following binary collisional ( $N_{\text{coll}}$ ) scaling. This result can be explained qualitatively as follows. In the classical Yang-Mills calculation the total multiplicity of gluons scales as  $Q_s^2$ , where  $Q_s$  is the dominant transverse momentum scale of the produced gluons, depending on both saturation scales  $Q_s^A$  and  $Q_s^B$ . The multiplicity of produced gluons  $\sim Q_s^2$  turns out to be roughly proportional to  $N_{\text{part}}$ . The energy density, on the other hand, scales as  $Q_s^3$ , and one expects it to scale as  $(N_{\text{part}})^\gamma$  with some  $\gamma > 1$ . It is therefore natural to expect the eccentricity in a saturation model to be larger than the traditional one following from  $N_{\text{part}}$ -scaling of the energy density. However, we see no reason in general for it to exactly mimic the result from  $N_{\text{coll}}$ -scaling.

In Fig. 3, we show a plot of the centrality dependence of the multiplicity for  $g^2\mu = 1.6$  GeV corresponding to an estimated gluon multiplicity of  $\sim 1000$  in central Au-Au collisions at RHIC <sup>2</sup>. The universal  $Q_s^2 \sim T_A$  prescription captures the observed centrality dependence of the multiplicity distribution. It has been argued [57] that in a realistic Monte Carlo implementation the KLN formalism can be recast in a form where the multiplicity is equivalent to one calculated from universal unintegrated gluon distributions. It appears unlikely however that this equivalence holds for other observables.

#### IV. CONCLUSIONS

We have shown in this brief note that the initial eccentricity of a relativistic heavy ion collision, computed in the Color Glass Condensate framework, is very sensitive to the transverse coordinate dependence of the saturation scale  $Q_s$ . When  $Q_s^2$  is proportional to the number of participants  $N_{\text{part}}$ , the energy density produced (near the

<sup>2</sup> The value  $g^2\mu = 1.6$  differs from the previous estimate of 2 GeV [16] primarily because these estimates had very low infrared cut-offs of order  $m \sim 1/R_A$ . For finite nuclei an infrared scale  $m$  of the order of the surface diffuseness of the Woods-Saxon density profile is required to regulate the Coulomb tails of the gluon field at large distances. While the dependence on  $m$  is weak, changing it by a factor of 10 does change the best estimate for  $g^2\mu$  by  $\sim 20\%$ .

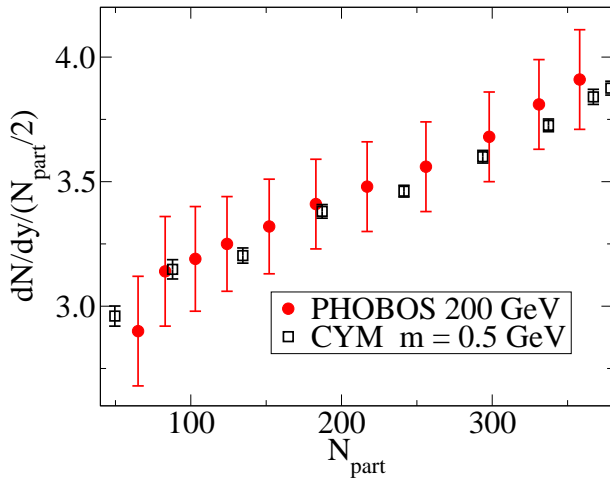


FIG. 3: The centrality dependence of the multiplicity. The closed circles are the PHOBOS data for the charged particle multiplicity per participant pair from Ref. [56]. The open squares are the CYM result for gluon multiplicity for  $g^2\mu = 1.6$  GeV and  $m = 0.5$  GeV and scaled by  $N_{\text{ch}}/N_{\text{tot}} \approx 2/3$ .

edge of one nucleus while near the center of the other) is small, leading to a large eccentricity. An argument based on the universality of the saturation scale leads to  $Q_s^2$  in nucleus A being proportional to the nuclear thickness function  $T_A$  of that nucleus alone. In this latter case, the saturation scale in the center of one nucleus does not depend on the other. While the two definitions have only logarithmic differences for multiplicity distributions, the energy density is larger and the eccentricity smaller in the universally scaling form of  $Q_s$  than for the non-universal case. In both cases, the result for the eccentricity is larger than the values typically employed in hydrodynamical models which assume ideal hydrodynamics. Conceptually, the universality of the saturation

scale is important because this universality is essential for a reliable framework to compute the properties of QCD at high energies.

The larger eccentricities from the CGC initial conditions are of great phenomenological interest because they allow for the possibility that there may be significant viscous effects in the quark gluon plasma. Our discussion should therefore be taken into account in future attempts to place an upper bound on the viscosity of the plasma. The reader should note however that these eccentricities were computed at very early times ( $\tau \approx 0.25$  fm) after the collision. After this time the eccentricity will immediately start to decrease because the edges of the system are expanding into the surrounding vacuum. Even if thermalization were to occur at the extremely early times of 0.6 fm (as some hydrodynamic models assume), the eccentricity will have decreased from the CYM values already at  $0.25 \text{ fm} \rightarrow 0.6 \text{ fm}$ , which is the time at which one needs the initial conditions. Therefore the initial eccentricities being assumed currently for hydrodynamic simulations may not too implausible leaving open the possibility that one indeed might have a “perfect fluid” [58]. A fuller understanding (and therefore a firmer bound on the viscosity) will require a dynamical understanding of how eccentricity decreases from the initial “Glasma” stage to the thermalized stage of the heavy ion collision.

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